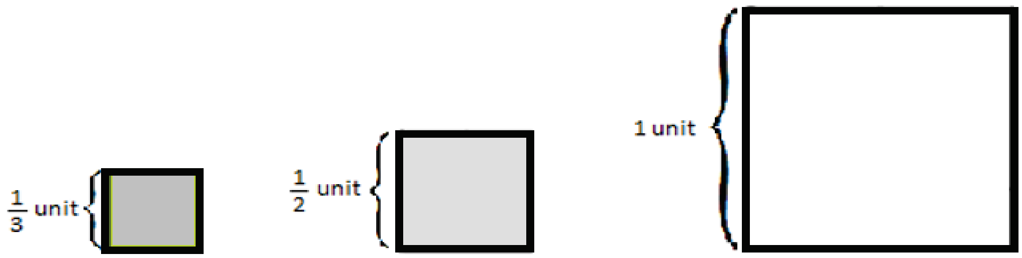
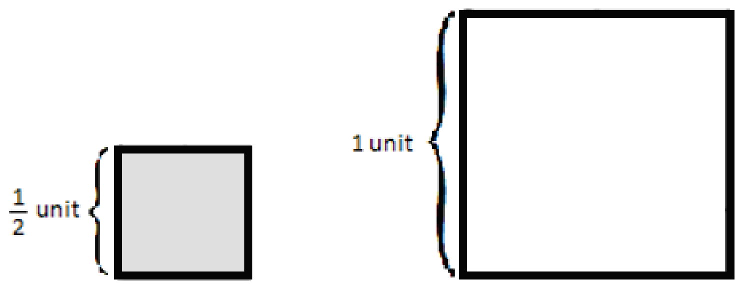
Additional Exercise from Scaffolding Box

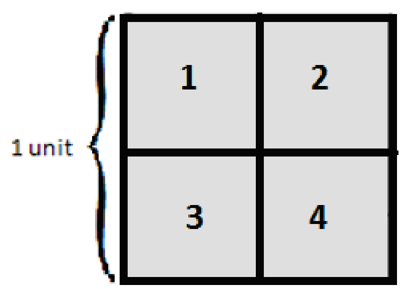
This is a sample activity that helps foster understanding of a cube with fractional edge length. It begins with three (two-dimensional) squares with side lengths of 1 unit, $\frac{1}{2}$ unit, and $\frac{1}{3}$ unit, which leads to understanding of three-dimensional cubes that have edge lengths of 1 unit, $\frac{1}{2}$ unit, and $\frac{1}{3}$ unit.



- How many squares with $\frac{1}{2}$ unit side lengths will fit in a square with 1 unit side lengths?

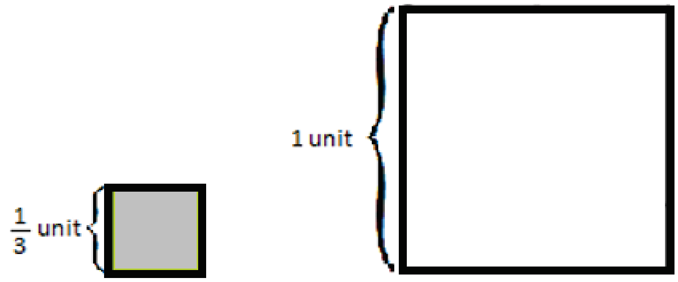


  - *Four squares with $\frac{1}{2}$ unit side lengths will fit in the square with 1 unit side lengths.*

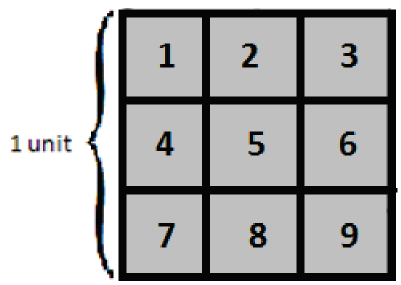


- What does this mean about the area of a square with $\frac{1}{2}$ unit side lengths?
  - *The area of a square with $\frac{1}{2}$ unit side lengths is $\frac{1}{4}$ of the area of a square with 1unit, so it has an area of $\frac{1}{4}$ square units.*

- How many squares with side lengths of $\frac{1}{3}$ units will fit in a square with side lengths 1 unit?



  - *Nine squares with side lengths of* $\frac{1}{3}$ *unit will fit in the square with side lengths of* 1 *unit.*



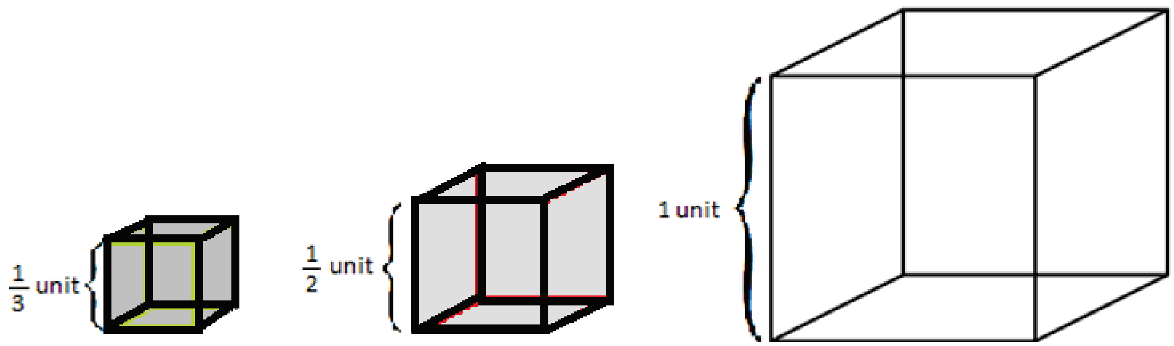
- What does this mean about the area of a square with $\frac{1}{3}$ unit side lengths?
  - *The area of a square with* $\frac{1}{3}$ *unit side lengths is* $\frac{1}{9}$ *of the area of a square with* 1 *unit side lengths, so it has an area of* $\frac{1}{9}$ *square units.*
- Let's look at what we've seen so far:

Side Length (units)	How many fit into a unit square?
1	1
$\frac{1}{2}$	4
$\frac{1}{3}$	9

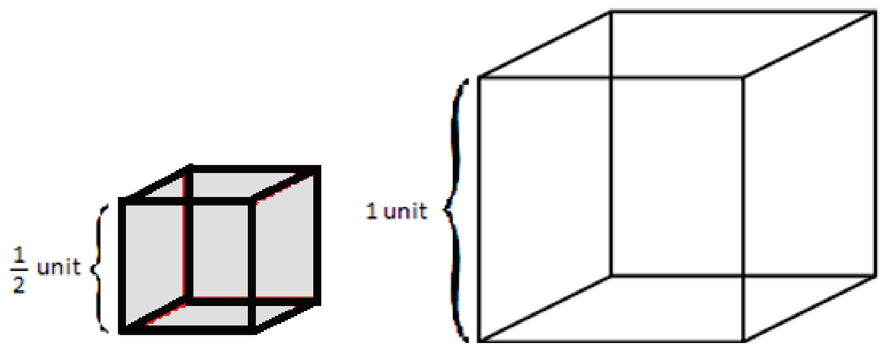
Sample questions to pose:

- Make a prediction about how many squares with $\frac{1}{4}$ unit side lengths will fit into a unit square; then draw a picture to justify your prediction.
  - 16 *squares*

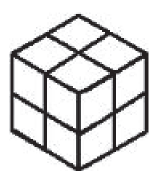
- How could you determine the number of $\frac{1}{2}$ unit side length squares that would cover a figure with an area of 15 square units? How many $\frac{1}{3}$ unit side length squares would cover the same figure?
    - *4 squares of $\frac{1}{2}$ unit side lengths fit in each 1 square unit. So if there are 15 square units, there will be* $15 \times 4 = 60$.
- Now let's see what happens when we consider cubes of 1 unit, $\frac{1}{2}$ unit, and $\frac{1}{3}$ unit side lengths.



- How many cubes with $\frac{1}{2}$ unit side lengths will fit in a cube with 1 unit side lengths?

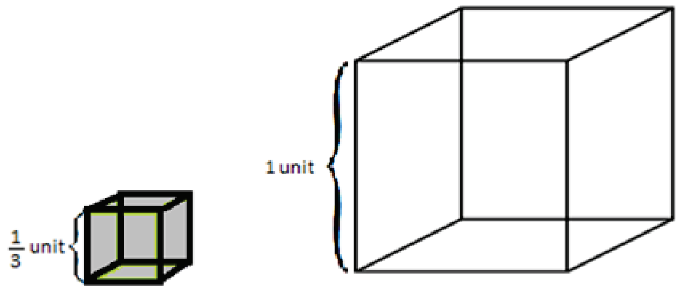


    - *Eight of the cubes with $\frac{1}{2}$ unit side lengths will fit into the cube with a 1 unit side length.*

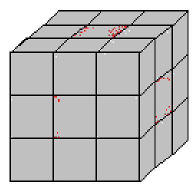


- What does this mean about the volume of a cube with $\frac{1}{2}$ unit side lengths?
    - *The volume of a cube with $\frac{1}{2}$ unit side lengths is $\frac{1}{8}$ of the volume of a cube with 1 unit side lengths, so it has a volume of $\frac{1}{8}$ cubic units.*

- How many cubes with $\frac{1}{3}$ unit side lengths will fit in a cube with 1 unit side lengths?



  - *27 of the cubes with $\frac{1}{3}$ unit side lengths will fit into the cube with 1 unit side lengths.*
- What does this mean about the volume of a cube with $\frac{1}{3}$ unit side lengths?
  - *The volume of a cube with $\frac{1}{3}$ unit side lengths is $\frac{1}{27}$ of the volume of a square with 1 unit, so it has a volume of $\frac{1}{27}$ cubic units.*



- Let's look at what we've seen so far:

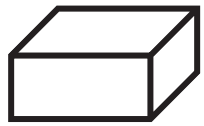
Side Length (units)	How many fit into a unit cube?
1	1
$\frac{1}{2}$	8
$\frac{1}{3}$	27

Sample questions to pose:

- Make a prediction about how many cubes with $\frac{1}{4}$ unit side lengths will fit into a unit cube, and then draw a picture to justify your prediction.
  - *64 cubes*
- How could you determine the number of $\frac{1}{2}$ unit side length cubes that would fill a figure with a volume of 15 cubic units? How many $\frac{1}{3}$ unit side length cubes would fill the same figure?
  - *8 cubes of $\frac{1}{2}$ unit fit in each 1 cubic unit. So if there are 15 cubic units, there will be 120 cubes because* $15 \times 8 = 120$.

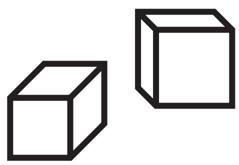
Understanding Volume

Volume



- Volume is the amount of space inside a three-dimensional figure.
- It is measured in cubic units.
- It is the number of cubic units needed to fill the inside of the figure.

Cubic Units



- Cubic units measure the same on all sides. A cubic centimeter is one centimeter on all sides; a cubic inch is one inch on all sides, etc.
- Cubic units can be shortened using the exponent 3.
  6 cubic cm $=$ 6 cm^3
- Different cubic units can be used to measure the volume of space figures – cubic inches, cubic yards, cubic centimeters, etc.